Internal Variability and Pattern Identification in Cirrus Cloud Structure within the Fokker-Planck Equation Framework

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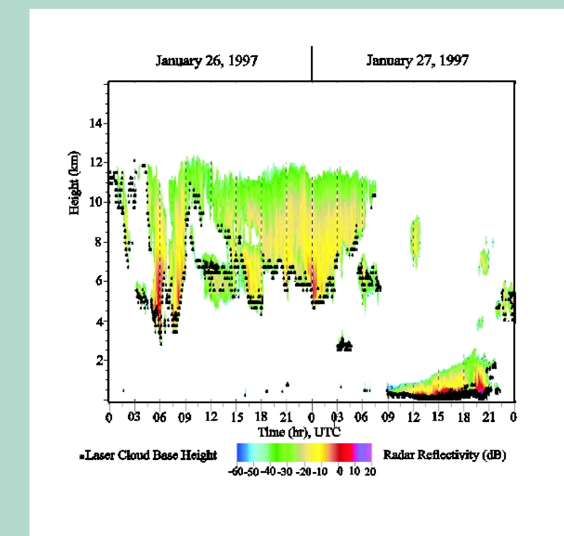
Objectives

Cirrus clouds play an important role in the climate system owing to their interwoven microphysical, dynamical, and radiative properties.

- The objective of this study is not only to characterize, but most importantly to quantify the different dynamical regimes in layers having different stratification defined on the basis of the thermodynamical properties of the cloud system.
- The backscattering cross-section signal η(t) is non-stationary, with highly irregular and clustered fluctuations owing to a set of various influences governing the particle motions at different temporal and spatial scales. The time-dependent tails of the probability distribution functions provide the signature of intermittency characterizing turbulent flows.
- Thus, it is the objective of this study to present a method for deriving an underlying mathematical or model-free equation --- the Fokker-Planck equation --- that governs the time-dependent probability distributions of the fluctuations at different delay times starting from observations of the backscattering cross-section.
- It is the objective of this study to distinguish and quantify the deterministic and stochastic influences on η(t) in cirrus clouds as described by the drift and diffusion coefficients of the Fokker-Planck equation.
- These coefficients characterize the dynamics of the processes in the layers having different stratification.

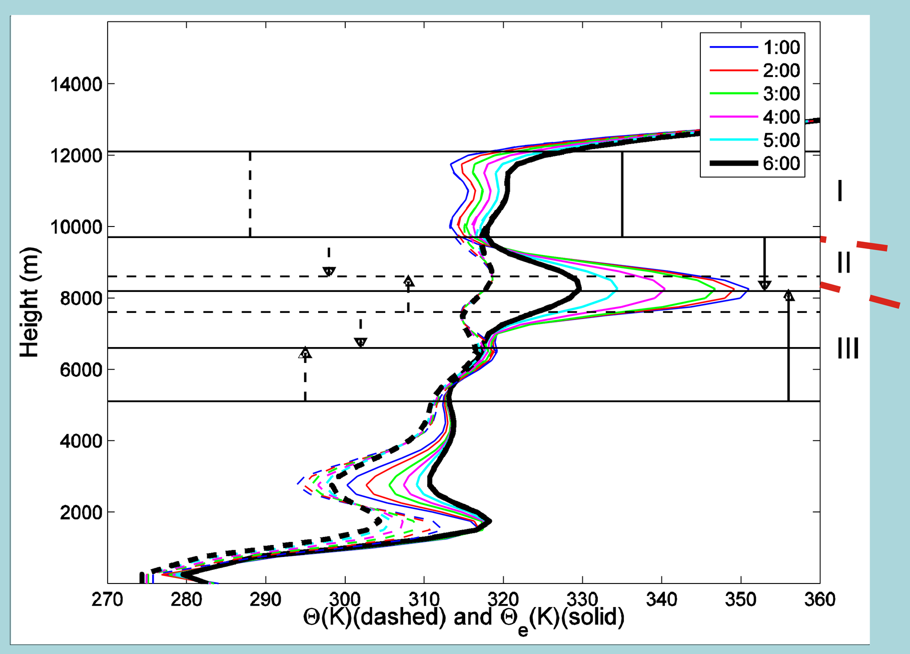
Data used



- A case study of cirrus cloud system observed on January 26 and 27, 1997 at the ARM Southern Great Plains site.
- 35 GHz MMCR backscattering cross section signal η(t)
- Radiosonde measurements of temperature, pressure, and ice/water mixing ratio.

Defining the stratification

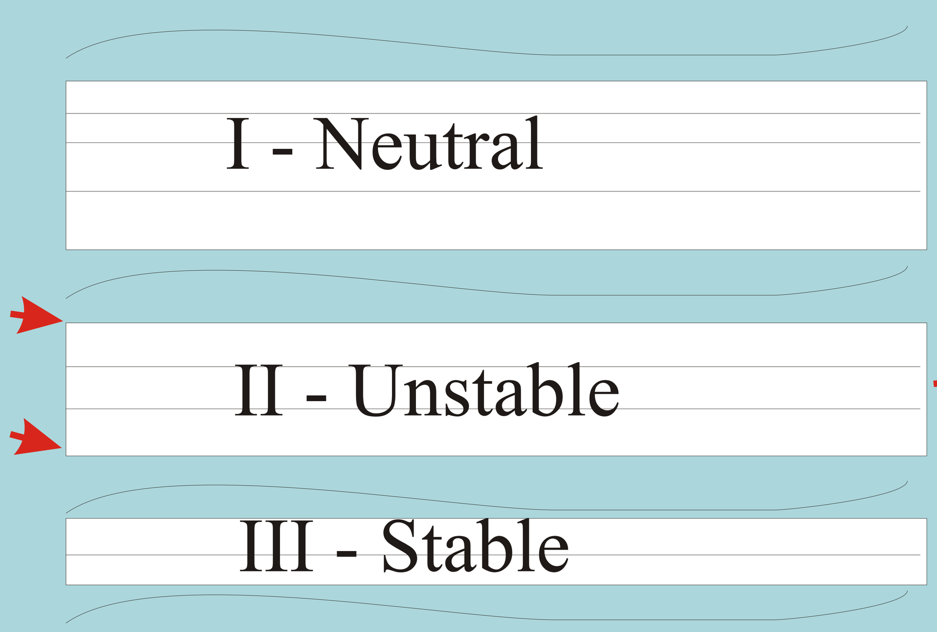
Based on the sign of the change in the potential temperature and equivalent potential temperature regions of neutral (I), unstable (II) and stable (III) stratification are identified.

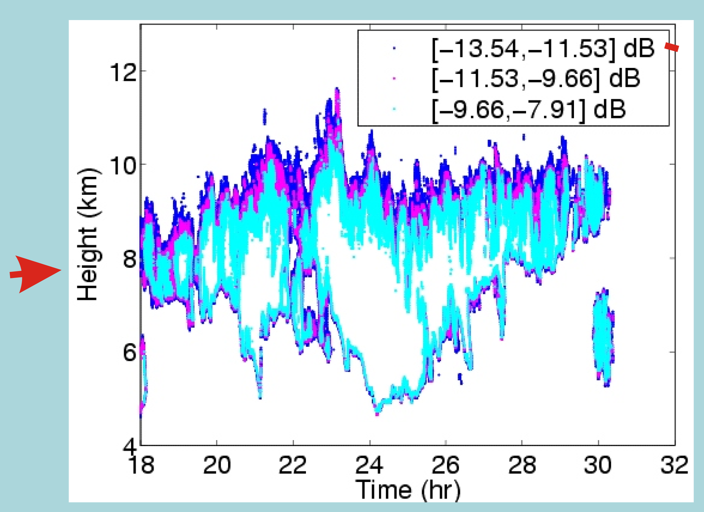


Defining the layers

Within each of the regions I, II, III there are a number of layers defined.

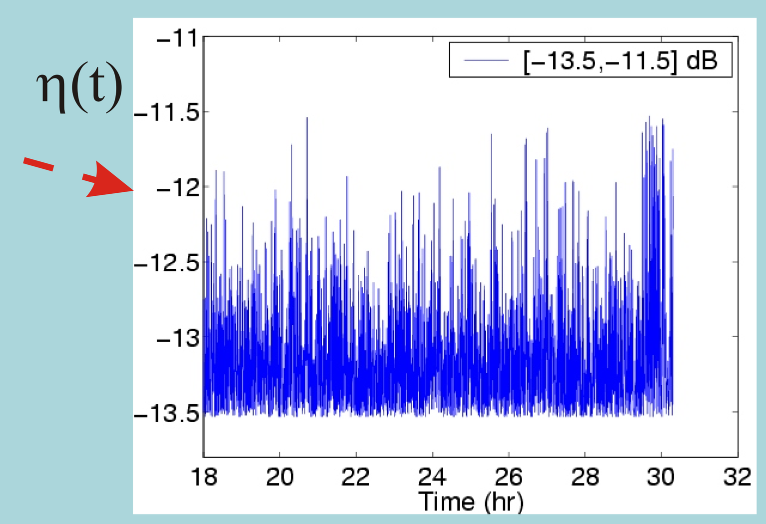
January 26 and 27, 1997





Defining the time series

Backscattering cross section signal η(t) at the maximum height in the sublayer [-13.5,-11.5] dB



Method of analysis - Fokker-Planck Equation approach

It is known that two equivalent master equations govern the dynamics of a system,

- the Fokker-Planck equation for the probability distribution function $p(\Delta x,\tau)$ of temporal and spatial signal increments
- and the Langevin equation, for the increments themselves $\Delta x(\Delta t)$.

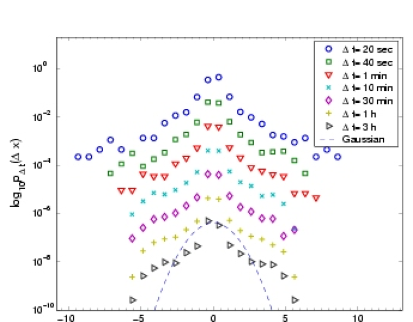


Figure 1: Probability distribution functions of the normalized increments of the backscattering cross-section signal $\eta(t)$ measured at h_{max} in the $[-25,-18.7]$ dB layer for different delay times Δt = 20 sec, 40 sec, 1 min, 10 min, 30 min, 1 h, and 3 h. The Gaussian distribution function (dashed curve) is shown for comparison.

Langevin Equation — for $\Delta x(\Delta t)$ (the increments of the signal)

$$\frac{d\Delta x}{d\Delta t} = D^{(1)}(\Delta x(\Delta t),\Delta t) + R(\Delta t)\sqrt{D^{(2)}(\Delta x(\Delta t),\Delta t)}$$

- $R(\Delta t)$ is a correlated noise with Gaussian statistics.
- The first term on the right side describes the deterministic part of the dynamics. The first term is called the drift term and it coincides with the *drift coefficient* $D^{(1)}$.
- The second term is proportional to the *diffusion coefficient* $D^{(2)}$ and describes the stochastic influences.
- In general, the Langevin equation for $\Delta x(\Delta t)$ and the Fokker-Planck equation for $p(\Delta x,\tau)$ are two equivalent differential equations that describe the dynamics of a system.

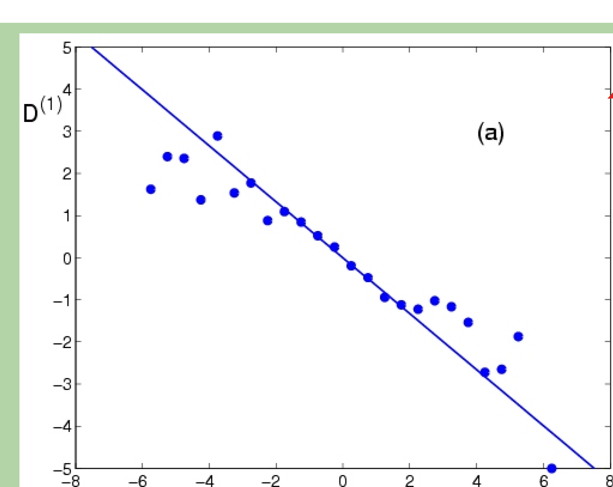
Fokker-Planck Equation — for $p(\Delta x,\tau)$ (the probability distribution function)

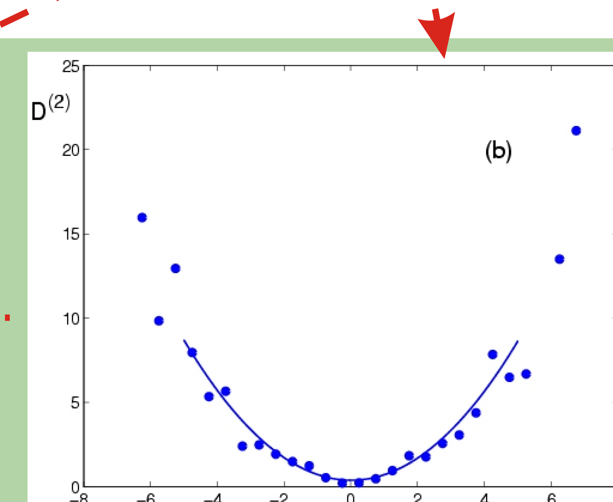
$$\frac{d}{d\tau}p(\Delta x,\tau) = [-\frac{\partial}{\partial\Delta x}D^{(1)}(\Delta x,\tau)+\frac{\partial^2}{\partial\Delta x^2}D^{(2)}(\Delta x,\tau)]p(\Delta x,\tau)$$

- $p(\Delta x,\tau)$ is the probability distribution function of the backscattering cross section signal.
- $D^{(1)}(\Delta x,\tau)$ and $D^{(2)}(\Delta x,\tau)$ are the drift and the diffusion terms that are the first two Kramers-Moyal coefficients $D^{(k)}(\Delta x,\tau)$.
- Thus the functional dependence of these drift and diffusion terms can be estimated directly from the moments of the conditional probability distributions involved in $D^{(k)}(\Delta x,\tau)$:

$$D^{(k)}(\Delta x,\tau) = \frac{1}{k!}\lim\frac{1}{\Delta\tau}\int(\Delta x'-\Delta x)^k p(\Delta x',\tau+\Delta\tau|\Delta x,\tau)d\Delta x'$$

for small $\Delta\tau$ and with $k = 1,2,\ldots$.





Functional dependences of $D^{(1)}$ and $D^{(2)}$

- The drift term $D^{(1)}(\Delta x)$ shows a linear dependence with Δx, and the diffusion term $D^{(2)}(\Delta x)$ can be approximated by a polynomial of degree two in Δx.
- Based on the functional dependences of $D^{(1)}$ and $D^{(2)}$, we find that the following approximations to the leading terms hold true:

$$D^{(1)}(\Delta x) \sim -\gamma\Delta x$$

$$D^{(2)}(\Delta x) \sim \beta\Delta x^2$$

$\gamma = 0.66$ and $\beta = 0.33$ in the $[-25,-18.7]$ dB range for neutral layer A.

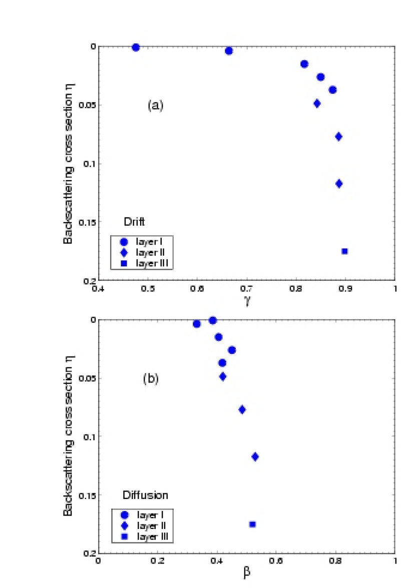


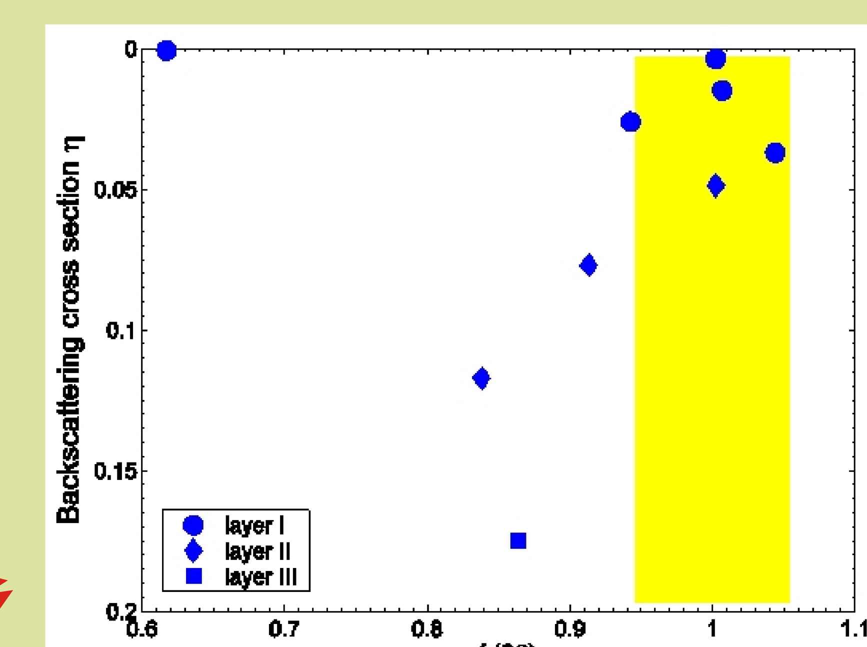
Figure 1: Values of the leading linear coefficient γ of the drift term ($D^{(1)}(\Delta x) \sim -\gamma\Delta x$) and the leading quadratic coefficient β of the diffusion term ($D^{(2)}(\Delta x) \sim \beta\Delta x^2$) of the backscattering cross-section η for the three major layers in the cirrus cloud. Each ordinate value represents the average value of η for each of the η time series for which $D^{(1)}$ and $D^{(2)}$ are obtained. The reverse order of the ordinate values is intended to give a sense of how the drift $D^{(1)}$ and diffusion $D^{(2)}$ terms change value with height.

References

Ivanova, K, H.N. Shirer, E.E., Clothiaux, 2006: Internal variability and pattern identification in cirrus cloud structure: Fokker-Planck equation approach, *J. Geophys. Res.*, **111**

Tatarskii, V.I., 2005: Use of the 4/5 Kolmogorov equation for describing some characteristics of fully developed turbulence, *Physics of Fluids*, **17**, 035110

Conclusions:



The internal variability of the cloud is revealed by the scaling properties of the backscattering cross-section signal η(t) inside each layer.

We obtain that the drift γ and diffusion β coefficients of the backscattering cross-section signal in the upper well-mixed, neutrally stratified layer of the cirrus cloud satisfy the relationship γ=2β, which is valid for fully developed turbulence owing to the Kolmogorov -4/5 law (Tatarskii, 2005).

Acknowledgments

This research was supported by the Office of Science (BER), U.S. Department of Energy, Grants No. DE-FG02-04ER63773